On the long-range dependence of spike trains

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Long-range dependence (LRD) has been observed in a variety of phenomena in nature, and for several years it has been known that successive interspike intervals (ISI) could be positively correlated over a large time-scale (see [Jac04] for a list of references), a phenomenon one could interpret as a memory effect in neuronal activity. Often, this is interpreted as originating from a non-Markovian system and can be related to other power-law characteristics of the spike train (see e.g. earlier works [Tei92]).

In a recent work [ROT17], we show numerically that a purely Markovian integrate-and-fire (IF) model, with a noisy slow adaptation term, can generate interspike intervals (ISIs) that appear as having LRD. However a proper analysis shows that this is not the case asymptotically. In fact, it is likely that any correlation between successive ISIs in a Markovian model will decay at least exponentially fast, as in the Ornstein-Uhlenbeck case.

For comparison, we consider a new model of individual IF neuron with fractional (non-Markovian) noise. Namely, we consider a stochastic process V that evolves from a resting value V_0 until it hits a threshold $S > V_0$, at which time the neuron fires and V is reset to its initial value. Between each jump, it follows this simple continuous dynamic driven by a fractional Brownian motion of Hurst parameter $H \in (\frac{1}{2}, 1)$. We study the correlations of the spike trains of this model. Our simulations prove that they have LRD, unlike classical IF models. Besides we may give a brief mathematical overview of hitting times problems for fractional dynamics, following [RT17].

On the other hand, to correctly measure long-range dependence, it is usually necessary to know if the data are stationary. Thus, a methodology to evaluate stationarity of the ISIs is presented and applied to the various IF models. We explain that Markovian IF models may seem to have LRD because of non-stationarities.

Finally, we may suggest how such long-range correlations can arise due to network activities of neurons.

References

[Jac04] B. S. Jackson. Including long-range dependence in integrate-and-fire models of the high interspike-interval variability of cortical neurons. *Neural computation*, 16(10):2125–2195, 2004.

[ROT17] A. Richard, P. Orio, and E. Tanré. An integrate-and-fire model to generate spike trains with long memory. *arXiv preprint arXiv:1702.03762 (*in revision at *Journal of Computational Neuroscience)*, 2017.

[RT17] A. Richard and D. Talay. Noise sensitivity of functionals of fractional brownian motion driven stochastic differential equations: Results and perspectives. In V. Panov, editor, *Modern Problems of Stochastic Analysis and Statistics*, pages 219–235, 2017.

[Tei92] M. C. Teich. Fractal neuronal firing patterns. In *Single neuron computation*, pages 589–625. Elsevier, 1992.