

# Phase dynamics of coupled neural oscillators: general principles of and differences between reduction techniques

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Rhythms and oscillatory behavior abound on all different scales of the human brain. Instrumental devices and recording techniques have been developed and refined to trace these dynamics, and meso- and macroscopic models have been designed to describe the mechanistic underpinnings of neural oscillations. However, linking recordings of brain activity to the underlying neuronal mechanisms is still one of the major challenges in neuroscience. This is especially true for large-scale brain dynamics, where networks of neural oscillators are commonly used to describe macroscopic behavior.

A vast amount of literature focuses on the network's phase dynamics in order to analyze synchronization properties that are believed to play an important role in information processing and functional connectivity within and across cortical areas. On a mesoscopic level one neural oscillator may follow a smooth limit cycle, which represents the fluctuations of the mean activity of a population of neurons. In this case, a phase variable can be easily defined and the network can be analyzed by considering the dynamic interactions of phase oscillators. In order to determine these phase dynamics, different phase reduction techniques can be used. But which of the many techniques is to be applied, crucially depends on the dynamical regime and the parameter region of the underlying neural oscillator model.

We compared the outcome of several mathematically sound phase reduction techniques applied to networks of weakly coupled Wilson-Cowan neural masses. While some reduction techniques only differ quantitatively, others yield qualitatively different phase models. By 'qualitatively different' we mean that the models display distinct stability properties. We highlight caveats and sensitive issues in the analytic derivation of phase models that may contribute to these dichotomies. We also illustrate the effects using numerical simulations. It appears that phase reduction techniques have to be tailored to the targeted macroscopic observable and the parameter regime under study. More importantly, though, we have to conclude that using heuristic phase models as guidelines for inferring neural network dynamics from data is challenging, if at all possible.