Testing for chaos in autonomous networks of spiking neurons

Claudius Gros(1), Rodrigo Echeveste(2)
(1)Institute for Theoretical Physics, Goethe University Frankfurt, Germany
(2)University of Cambridge, Cambridge, United Kingdom

Networks of autonomously spiking or oscillatory neurons are central to dynamical neurosciences [1]. These networks can be composed of either spontaneously active neurons or of excitable neurons, the biological case, which enter a spiking limit cycle only for strong enough steady inputs [2]. Of interest are in particular phases in which subsets of synchronized neurons coexist with neurons that are not phase locked, as it occurs also within the Kuramoto model for distributed natural frequencies. Excitable neurons do not however dispose of a natural frequency, which is induced in turn by the self-organized network activity through the mean input current.

Drifting neurons can be regarded, in phases in which synchronized and drifting neurons coexist, as self-organized oscillators that are driven periodically by the subset of synchronized neurons. Chaotic activity may arise and be tested for using classical methods [3]. We propose here, alternatively, to use Takens embedding theorem, which states that a chaotic attractor of dimension D may be sampled via tuples of D measurements of a selected variable. We find that the interspike intervals of a single neuron is in this context an intuitive variable. Plotting pairs of consecutive interspike intervals provides in particular for an economic test that allows to distinguish with the naked eye between limit-cycle and chaotic dynamics and to evaluate projected fractional dimensions.

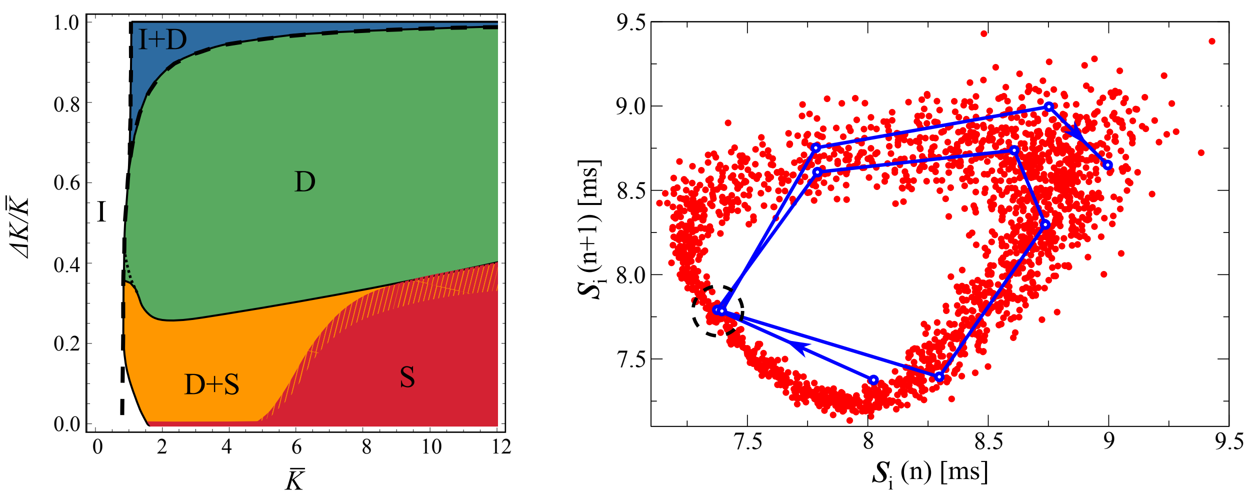

Figure 1: (left) The phase diagram of a network of 100 all-to-all connected excitable neurons, with individual synaptic couplings that are draw uniformly from within $[\overline{K}-\Delta K, \overline{K}+\Delta K]$. Apart from the inactive (I) phase, which dominates for low average coupling strengths $\overline{K}$, there are drifting (D) and synchronized (S) phases, in part coexisting.
(right) Collection of two consecutive spike intervals $S_i(n)$ and $S_i(n+1)$, for a drifting neuron i in the D+S phase (red dots; $\overline{K}=2$ and $\delta K=0.2\,\overline{K}$). The blue line connects a selection of consecutive pairs $(S_i(n), S_i(n+1))$, with the dashed circle (black) drawing attention to a near periodic event. The Hausdorff dimension of ~ 1.8 of the pairs of interspike intervals indicates the presence of chaotic dynamics.

References

[1] P. Ashwin, S. Coombes, R. Nicks. Mathematical frameworks for oscillatory network dynamics in neuroscience. *The Journal of Mathematical Neuroscience* **6**, 2 (2016).

[2] R. Echeveste, C. Gros. Drifting states and synchronization induced chaos in autonomous networks of excitable neurons. *Frontiers in Computational Neuroscience* **10**, 98 (2016).

[3] H. Wernecke, B. Sándor, C. Gros. How to test for partially predictable chaos. *Scientific Reports* **7**,1087 (2017).