Kuramoto model for excitation-inhibition-based oscillations

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The Kuramoto model is an idealized mathematical model for exploring the birth of collective synchronization in its most simple form. It consists of a population of heterogeneous, all-to-all coupled oscillators, and is a unique example of exactly solvable system of nonlinear differential equations [1, 2]. Yet, the Kuramoto model was originally not intended as a literal description of any specific system, and only finds limited applications in the modeling and analysis of natural oscillatory phenomena.

An important example of collective synchronization are neuronal oscillations. Despite continued work using the Kuramoto model to investigate large-scale neuronal rhythms, it remains unexplored whether the model actually accounts for the neuronal mechanisms resulting in such oscillations. Here we derive a simple, two-population Kuramoto model, that describes a fundamental mechanism for the generation of large scale brain oscillations: The feedback loop between fast excitation and slow inhibition, often referred to as as PING, pyramidal-interneuron gamma rhythms, in large neuronal networks [3, 4].

As the original Kuramoto model, the Excitation-Inhibition Kuramoto model is analytically solvable, and accurately describes the main features of the EI-based rhythms: (i) In absence of recurrent connections, oscillations emerge when excitatory neurons are faster than inhibitory neurons. (ii) Otherwise, when inhibition is faster than excitation, strong enough recurrent-coupling is necessary for oscillations to emerge. (iii) Excitation always precedes inhibition. (iv) The transition between incoherence and synchronization is often sub-critical.

References

[1] Y. Kuramoto. *Chemical Oscillations, Waves, and Turbulence.* Springer-Verlag, Berlin, 1984.

[2] S. H. Strogatz. From Kuramoto to Crawford: exploring the onset of synchronization in populations of coupled oscillators. *Physica D*, 143:1–20, 2000.

[3] M.A. Whittington, R.D. Traub, N. Kopell, B. Ermentrout, and E.H. Buhl. Inhibition-based rhythms: experimental and mathematical observations on network dynamics. *Int. Journal of Psychophysiol.*, 38(3):315 – 336, 2000.

[4] Xiao-Jing Wang. Neurophysiological and computational principles of cortical rhythms in cognition. *Physiological reviews*, 90(3):1195–1268, 2010.