Stochastic strategies to include memory in neuronal models

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Neuronal dynamics have been widely investigated in the last decades by means of the powerfull mathematical tool such as that of stochastic processes, but many features are still unknown. One of the main models for such dynamics is the stochastic Leaky Integrate-and-Fire (sLIF) model with solution the well-known Ornstein-Uhlenbeck (OU), whose markovian nature does not match the need to include memory in neuronal models ([1]). For this reason, as a first step, some modified sLIF models were suitably realized by using time-inhomogeneous Gauss-Markov (GM) processes ([2]). Theoretical results on the first passage times (fpt) densities of these processes can be well exploited to model firing activity and rate. Recently, some phenomena, among them multi-time-scale dynamics and adaptation ([3], [4]), led us to build models with memory by which more physiological evidences can be better described. To this purpose, further strategies have been specialized or ad hoc devised. Some of these strategies provide stochastic models that can be essentially arranged in three different categories: *(i)* sLIF models with jumps, *(ii)* sLIF models with colored noise, *(iii)* fractional sLIF models. The models of the first category turn out to be particularly useful to describe the neuronal activity of coupled neurons ([5]) or of linked neurons in a network ([6]), whose activities are modified whenever interaction stimuli occur. The models of the second category are suitable to include a correlated input noise ([7]) in sLIF models ([8]) so that the input itself preserves memory. In the category of fractional sLIF models, the dynamics of a neuronal potential keeping in memory its previous evolution ([8],[9]) can be modeled by means of a time-fractional Langevin equation ([10]). In the same category, we can also consider the model based on a Stochastic Differential Equation (SDE) driven by a fractional Gaussian noise ([11]). In this model, under some assumptions on the input stimuli and the Hurst index of the noise, heavy tailed auto-covariance functions can represent a kind of long memory of the modeling processes. The stochastic methodologies adopted to investigate the above models are based on approximating Gauss-Markov processes for *(i)*, integrated Gaussian processes for *(ii)*, fractional time-derivatives and the fractional Brownian motion for *(iii)*, respectively. Simulation results of the above models will be provided, compared and discussed.

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