Hawkes processes as a spike train model

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A spike train is the set of firing times of a given neuron. Hence the most simple class of stochastic model we can use to describe spike trains is the class of temporal point processes, that are random sets of times. Since the early work of Chornoboy et al. [5], Hawkes processes have been extensively studied with application to neurosciences in mind due to their relevant, yet rather simple, way to mimic synaptic integration. For instance, for a network of neurons indexed by i from 1 to n, the model is based on interaction variables of the form

$$X^i(t) := \sum_{j=1}^{n} \int_0^{t-} h_{j\to i}(t-t') N^j(dt') = \sum_{j=1}^{n} \sum_{\tau_k^j < t} h_{j\to i}(t-\tau_k^j),$$

where the τ_k^j's are the firing times of neuron j. The interaction function $h_{j\to i}$ characterizes the influence of the spikes of neuron j onto the firing rate of neuron i: the synaptic strength and the transmission delay are encoded in the magnitude and the shape of the function.

We propose a review on this model and especially on its several mean-field limits, showing how Hawkes processes can be viewed as the individual-based/microscopic stochastic model associated with several macroscopic deterministic models:

- age structured or time elapsed model [2, 8]. It is based on the age variable that counts the time elapsed since the last discharge to take into account the refractory period. Synchronization via oscillatory behavior and stimulus sensitivity of the mean-field system [4] will be examined.
- monotone cyclic feedback systems [6]. It is based on the classification of neurons into several sub-populations. The interaction function $h_{j\to i}$ depends on the populations where neurons i and j lie. These systems show oscillatory behavior and can be viewed as spatial discretization of neural fields.
- neural fields [1, 3]. It is based on spatial state variables. The interaction function $h_{j\to i}$ depends on the locations of neurons i and j. Numerical results show that finite size effects can deny propagation of neural activity in opposition to what is predicted by the neural field equation.

References

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