Temporally varying neural responses to spatially periodic stimuli

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Certain images that have spatial components in a narrow band of wave numbers have been shown to induce temporally varying neural responses. In pattern sensitive epilepsy, striped lines can trigger epileptic seizures if they are close to 3 cycles deg^{-1} (cpd) [2]. Similarly, images – including abstract artwork – with peaks in power near 3 cpd are known to cause aversion in healthy individuals [1]. Both of these phenomena have been shown to induce abnormal temporal activity in electroencephalography (EEG) or magnetoencephalograhy (MEG) recordings.

Neural fields have proven useful at modeling the spatiotemporal dynamics of ensembles of neurons and capturing many experimentally observed patterns, such as planar and spiral waves [3]. We are thus motivated to consider a spatially extended neural field model where a static, spatially periodic stimulus is provided as input to the excitatory and inhibitory neural populations. By adjusting system parameters such as the amount of recurrent excitation, we may place the stimulus-free system near a so-called Turing-Hopf bifurcation, where the uniform steady state is spontaneously lost to temporally and spatially periodic patterns with wave number m^*. Simulations and numerical bifurcation analysis for the 1-D system demonstrate the desired resonance, displaying spatially periodic temporal oscillations with very weak stimuli for some wave numbers while requiring much stronger stimuli for others. We analytically show that a weak stimulus with wave number m^* destabilizes the steady state, and find the stability boundary as a function of the recurrent excitation and stimulus strength. Finally, we present a more realistic 2-D system simulated on a GPU that exhibits this strong sensitivity to the spatial frequency of the stimulus. These 2-D simulations also allow us to demonstrate resonance to noisy images with dominant wave numbers near m^*, matching experimental findings in visual discomfort.

To our knowledge, no computational neuronal model has been proposed to account for these striking spatial resonance features. We consider the following neural field model in one spatial dimension x:

$$\begin{aligned}
\frac{\partial u(x,t)}{\partial t} &= -u(x,t) + f(J_{ee} * u(x,t) - J_{ie} * v(x,t) - \theta_e + q\ S(x;m)) \\
\tau\ \frac{\partial v(x,t)}{\partial t} &= -v(x,t) + f(J_{ei} * u(x,t) - J_{ii} * v(x,t) - \theta_i + q \cdot r\ S(x;m)),
\end{aligned}$$

where u and v, the excitatory and inhibitory neural populations, respectively, provide a low-pass filter on the firing rate, reflecting the neurons' lower-frequency behaviors. The timescale of the inhibitory population is given by τ, while $J_{\alpha\beta} = a_{\alpha\beta} K_\alpha$, and $a_{\alpha\beta}$ are the maximum connection strengths from α to β. $K_{e,i} = \frac{1}{\sigma_{e,i}\sqrt{2\pi}}\ \exp\left(\frac{-|x|^2}{{\sigma_{e,i}}^2}\right)$ is a gaussian kernel with lateral inhibition, and "$*$" denotes spatial convolution over the entire domain. Each component has a threshold, $\theta_{e,i}$, that helps determine its excitability. $S(x;m)$ is the spatially periodic, time-independent stimulus, which generally takes the form $\cos(\frac{2\pi\, m\, x}{N})$, where N is the number of neuronal populations being modeled, and the amplitude of the stimulus is given either by q for $u(x,t)$ or $q \cdot r$ for $v(x,t)$. The firing rate, or transfer, function f is the sigmoidal logistic function, $f(u) = \frac{1}{1+\exp(-4u)}$.

Using simulations and numerical analysis tools, we have found that the system exhibits a strong sensitivity to stimuli with spatial frequencies near m^*, and less or no sensitivity to other spatial frequencies in both one and two spatial dimensions. Surprisingly, the same resonant behavior observed in this large spatial system can also be observed in a minimal system of two populations (2 excitatory and 2 inhibitory). We are further able to demonstrate the loss of stability of the steady state through a perturbation calculation, allowing us to analytically determine the stability boundary for small-amplitude stimuli in the minimal system. Our calculation shows very good agreement with the numerically computed boundary.

Our results suggest that different parameter values involving, e.g., connectivity strengths, could cause neuronal networks to exhibit a natural sensitivity to particular spatial frequencies. The spatially-resonant behaviors observed in pattern-sensitive epilepsy and with aversive images may then be explained within the framework of dynamical systems, as spatially periodic stimuli with components near the resonant wave number push the system past a symmetry-breaking bifurcation into a spatio-temporal pattern forming regime.

References

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