Integrate and Fire like models with stable distribution for the Interspike Intervals.

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In 1964, Gernstein and Mandelbrot proposed the Integrate and Fire model to account for the observed stable behavior of the Interspike Interval distribution. Their study of histograms of ISIs revealed the stable property and they suggested modeling the membrane potential through a Wiener process in order to get the inverse Gaussian as first passage time distribution, i.e. a stable distribution. Holden (1975) observed that stable distributions determine a simple transmission pathway.

Later many variants of the original model appeared with the aim to improve its realism but meanwhile researches forgot the initial clue for the model. The Leaky Integrate and Fire model that has not stable FPT distribution gives an example. The same holds for many other variants of this model. More recently Persi et al.(2004) studying synchronization patterns, proposed a time non homogeneous integrate and fire model accounting for heavy tail distributions. The existence of heavy tails, typical of stable distributions is well recognized in the literature (see for example Tsubo et al., Gal and Morom and references cited therin).

Signals from different neurons are super-imposed during the elaboration. Lindner (2006) [7] showed inconsistencies in the generally accepted hypothesis on a resulting Poissonian distribution. However, this is the basic assumption of classical Leaky Integrate and Fire models. Different ISIs distributions would determine an incredible variety of firing distributions as the information progresses in the network. Hence, it seems unrealistic to admit ISIs that cannot reproduce the same distribution when summed. This suggest the development of Integrate and Fire models using stable distributions. Furthermore, the stable ISIs paradigm gives rise to a more robust transmission algorithm since a possible lack of detection of some spike from the surrounding neurons does not change the nature of the final distribution.

Here we rethink to the problem, taking advantage of the mathematical progresses on Lvy processes (see Kyprianou, 2014) Hence, we propose to start the model formulation from the main property, i.e. the stable nature of the ISIs distribution. We follow the Integrate and Fire paradigm but we model the membrane potential through a randomized random walk whose jumps are separated by inter-times with stable distribution (or in the domain of attraction of such distribution). Observing that the supremum of the modeled membrane potential results to be the inverse of a stable subordinator allows determining the Laplace transform of the ISIs distribution. This is a preliminary contribution since we limit ourselves to some aspects of the modelling proposal ignoring any attempt to fit data. We are conscious that these are preliminary results and some further mathematical study will be necessary to obtain models fitting data.

References

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