Neural networks do not become asynchronous in the large size limit: there is no propagation of chaos

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Motivations:
The mathematical description and analysis of the thermodynamic limit of neuronal networks is important because it can shed light on the collective behaviors of large assemblies of neurons such as those which are found in the brains of mammals. There are several other reasons why it is worth investigating these questions. One is that the limit equations, albeit complicated, are much more concise than the collection of equations describing individual neurons in the network. This makes their analysis possible with the consequence that general laws can be formulated rigourously. Another reason is that the mathematical framework in which they are derived allows one to evaluate precisely under which hypotheses the results are valid and how changes in the hypotheses will impact them.

Previous Work:
This area of research has primarily been investigated using methods from theoretical physics as in the work of H. Sompolinsky (field theory) (SCS88), N. Brunel (BH99) (Master equation, population based approaches), among many others. Only recently has this problem received attention from mathematicians. Building on previous work on the large deviations of weakly interacting diffusions, in particular that of A. Guionnet (Gui97), O. Moynot and M. Samuelides (MS02) have generalized the results of Sompolinsky et al. for rate neurons, but the fact is that the vast majority of these approaches assume that the synaptic weights are either known and constant or that, if they are random, they are statistically independent.

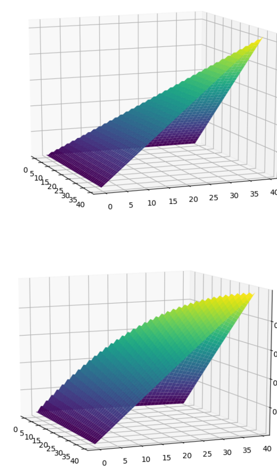

Figure 1: Time correlations in the **thermodynamic limit**. Top: of a given neuron. Bottom: between two neighbouring neurons. **Neurons activities are correlated.**

Our contribution:
We have developed a new method for establishing the thermodynamic limit of a network of fully connected rate neurons with correlated, Gaussian distributed, synaptic weights, and random inputs. The method is based on the formulation of a large deviation principle (LDP) for the probability distribution of the neuronal activity of a sequence of networks of increasing sizes. The motivation for using random connections comes from the fact that connections in neural networks are complex, poorly known and heterogeneous. The motivation for introducing correlation is the emphasis in computational modelling of neuroscience that neural networks are modular, and the correlations in the connection distribution reproduce this modularity, unlike in (MS02; BFT15). The limiting probability law is Gaussian and its mean and covariance functions are computed using a very quickly converging fixed point algorithm. Our outstanding new result is the fact that, unlike in all previous works (SCS88; MS02) in the thermodynamic limit the network does not become asynchronous, there is no propagation of chaos: neurons remain correlated and the amount of correlation can be described precisely from the correlation between the synaptic weights.

References

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