Explaining the hovering stochastic oscillations in self-organized quasi-critical systems

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Self-organized criticality (SOC) with conservative systems is by now an understood subject, fully integrated to the Statistical Mechanics of non-equilibrium systems. But, for dissipative systems, the situation is not clear and several open problems remain. Models of dissipative systems (earthquakes, forest fires, neuronal avalanches) have been proposed since the 90´s with the claim that they are examples of true SOC systems. The present consensus is that they are not: even if one can propose slow drive mechanisms to counterbalance dissipation, so that the system is at least conservative in average, this needs some fine tuning in the hyperparameters of these adaptive mechanisms. Without this fine tuning, such models present stochastic oscillations that hovers near the critical point and do not disappear in the thermodynamic limit, what has been called self-organized quasi-criticality (SOqC) [1] and self-organized super-criticality [2,3,4]. Here we explain the origin of these stochastic oscillations as an outcome of a Neimark-Sacker bifurcation from a stable spiral to an indifferent one at the critical point. The stochastic hovering oscillations near the critical point turns out only stochastic hovering (without well defined period) exactly at the critical point. This scenario seems to solve a 20 years open problem in the area. Our results are obtained with a mean-field analysis of networks of simple discrete-time stochastic neurons. We found that the use of such elements is essential to obtain simple and transparent analytic results. We also found a mathematical connection between the hovering oscillations of our neuronal network and stochastic oscillations in discrete-time predator-prey systems.

References

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