Homoclinic dissection of fold/hom neuron bursting models

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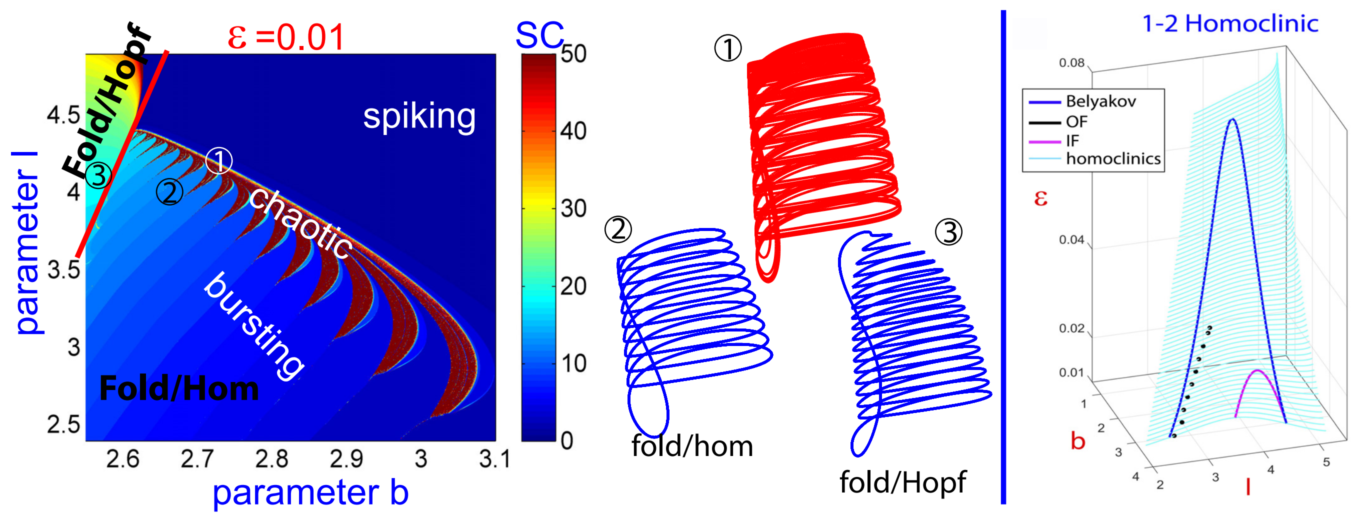
The wide-range assessment of brain and behaviors is one of the pivotal challenges of this century. To understand how an incredibly sophisticated system such as the brain *per se* functions dynamically, it is imperative to study the dynamics of its constitutive elements – neurons. Fold/hom bursting phenomena is found in numerous fast-slow models, and specially in neuron models [5, 6]. The existence of an homoclinic bifurcation curve is a requirement to its appearance, but the real structure of the homoclinic bifurcations in the global parametric space of fold/hom neuron models is not known. Here we provide a global analysis of the organization of the homoclinic bifurcation manifolds in the parametric phase space [2, 3] and how topologically different 2D manifolds are present. The detailed numerical analysis and continuation curves give results that lead us to conjecture the theoretical organization of these systems (exemplified in the canonical Hindmarsh-Rose neuron model). All the different homoclinic bifurcation manifolds are classified and several codimension-2 homoclinic bifurcation curves are shown (like Orbit-flip, Inclination-flip and Belyakov points). The global picture reveals several codimension-3 points that are detailed. Besides, due to the structure of the homoclinic manifolds as tubular-like shapes with very sharp folds, isolas [1] of homoclinic bifurcation curves are shown once the small parameter is fixed on the model. Moreover, isolas of codimension-2 homoclinic curves are detected, like isolas of Belyakov points. All these bifurcations are connected with the spike-adding process and canards in fast-slow models as each spike-adding bifurcation is related with the existence of one homoclinic bifurcation manifold that is exponentially close to the rest of homoclinic bifurcation manifolds, giving rise to a homoclinic structure that we call "Homoclinic mille-feuille" [3].



The present analysis completes previous partial analysis of fold/hom neuron bursters [4, 6].

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