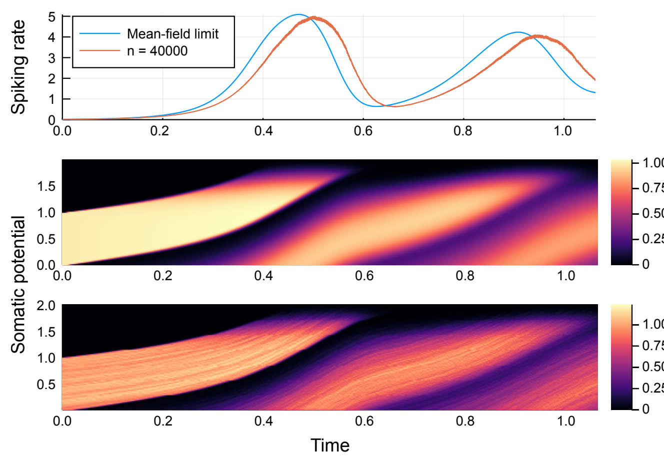
On a toy network of neurons interacting through nonlinear dendritic compartment

Nicolas Fournier, Etienne Tanré and Romain Veltz

The dendrites of many neurons are endowed with active mechanisms which confer them properties of excitability and enable the genesis of local dendritic spikes. In this work, we consider the propagation of dendritic spikes in a dendrite composed of a single branch. These local dendritic spikes are due to voltage dependent ion channels (i.e. sodium, calcium or NDMA spikes). Because the dendrite morphology is modeled after a half line, dendritic spikes propagate in both directions, although with possibly different speeds. Two dendritic spikes propagating in opposite directions will cancel out when they collide as in the case of the axon because of the refractory period, some ion channels becoming inactivated after a spike.

We focus on an abstract description of this nonlinear behavior which is more amenable to analysis. This description reveals a rich mathematical structure that we study through the use of combinatorics. This also provide an algorithm for an efficient simulation. In passing, we link this description to the famous Ulam problem [Ulam, 1961, Hammersley, 1972] opening the door for a mean field model.

We then consider a large number n of neurons, each being connected to approximately N other ones, chosen at random. When a neuron spikes, which occurs randomly at some rate λ depending on its electric potential, its potential is set to a minimum value v_{min}, and this produces, after a small delay, two fronts on the dendrites of all the neurons to which it is connected. Fronts move at constant speed. When two fronts (on the dendrite of the same neuron) collide, they annihilate. When a front hits the soma of a neuron, its potential is increased by a small value w_n. Between jumps, the potentials of the neurons are assumed to drift in $[v_{min}, \infty)$, according to some well-posed ODE. We prove the existence and uniqueness of a heuristically derived mean-field limit of the system when $n, N \to \infty$ with $w_n \simeq N^{-1/2}$. We make use of some recent versions of the results of Deuschel and Zeitouni [Deuschel and Zeitouni, 1995] concerning the size of the longest increasing subsequence of an i.i.d. collection of points in the plan. We also study, in a very particular case, a slightly different model where the neurons spike when their potential reach some maximum value v_{max}, and find an explicit formula for the (heuristic) mean-field limit.



Intensive numerical simulations are presented for cases not covered by our mathematical results as shown in the figure where we plot on the first picture the maps $t \mapsto n^{-1} \sum_{i=1}^{n} \lambda(V_t^{i,n})$, for the finite size network with $n = 40000$ neurons, as well as $t \mapsto \mathbb{E}[\lambda(V_t)]$, for $(V_t)_{t \geq 0}$ the unique solution to the nonlinear SDE, the mean-field limit. The second picture represents $(g(t, v))_{t \geq 0, v \geq 0}$, where $g(t, \cdot)$ is the density of the law of V_t. The third picture represents the empirical distribution of somatic potentials for $n = 40000$.

This is one of the first work on mean field limits of networks of spiking neurons with a dendritic branch.

References

[Deuschel and Zeitouni, 1995] Deuschel, J.-D. and Zeitouni, O. (1995). Limiting Curves for I.I.D. Records. *The Annals of Probability*, 23(2):852–878.

[Hammersley, 1972] Hammersley, J. M. (1972). A few seedlings of research. The Regents of the University of California.

[Ulam, 1961] Ulam (1961). Monte Carlo calculations in problems of mathematical physics.